PROGRAM OF ENTRANCE TEST IN MATHEMATICS AND COMBINATORICS FOR STUDENTS ENTERING MASTER PROGRAM

Regulations

The entrance test is a written exam, consisting of 6-10 problems/tasks of different difficulty levels. All problems require a complete solution with a detailed proof/explanation. The following resources are allowed during the exam:

- Wikipedia.org
- Wolframalpha.com
- Live.sympy.org
- Python.org/shell
- CoCalc (SageMath)

Counting an answer using one of these systems is not a complete solution or a proof. These resources can only be used as a hint.

To complete the tasks given 3 astronomical hours.

Foundations of algebra

1. Groups, Abelian groups, normal subgroups, classical examples: groups of numbers by addition and multiplication, a group of non-degenerate matrices, a group of permutations, a group of residues by addition and multiplication.
2. Rings and commutative associative rings with unit. Examples: rings of numbers, rings of matrices, rings of residues, rings of polynomials.
3. Fields, definition and examples: the field of rational numbers, real numbers, complex numbers.

Elements of linear algebra

1. Systems of linear equations and the Gaussian elimination.
2. Vector spaces. Definition, examples: a space of rows, spaces of square matrices, spaces of symmetric and skew-symmetric square matrices, spaces of polynomials of one variable.
3. Linearly independent and linearly dependent systems of vectors.
4. A basis and the dimension of a vector space.

Basics of calculus

1. Sequences. Limits of sequences. Examples of convergent and divergent sequences.
2. Continuous functions of one variable. Limits of functions.
3. Derivative. Differentiable functions. Mean value theorems: Fermat, Roll, Lagrange, Cauchy.
4. Infinitely small and limited quantities. Big-O notation.
5. Taylor series.

6. Indefinite integrals. Antiderivative.
7. Definite integrals. Improper integrals.

Foundations of topology in R^n

1. The topology of the real line. Intervals and segments. Convergent subsegments. Open and closed sets.
2. Open and closed sets in a multidimensional space.
3. Continuous maps.
4. Compact subsets in R^n: finite subcovers. Closure and boundedness.

Combinatorics and probability

1. Pigeonhole (Dirichlet's box) principle (with 2-3 examples of its application).
2. Standard counting rules: the rule of sum and the rule of product.
3. Combinations, placements and permutations.
4. Newton's binomial theorem.
5. Elementary events and finite sample spaces. The classical definition of probability. Computation of probabilities in classical settings.
6. Graphs: definitions. Complete graphs, simple graphs, trees, cycles. Degrees of vertices.

References

1. E.B. Vinberg, «A Course in Algebra», Graduate Studies in Mathematics, AMS, Vol. 56, 2003.
2. V.A. Zorich, «Mathematical Analysis I», Springer-Verlag Berlin Heidelberg, 2004.
3. L.B. Koralov, Ya.G. Sinai, «Theory of Probability and Random Processes», Springer-Verlag Berlin Heidelberg, 2007.
4. W. Rudin, «Principles of Mathematical Analysis», International Series in Pure and Applied Mathematics, McGraw-Hill Education, 1976, 3rd Edition.
5. R. Stanley, «Enumerative Combinatorics», Cambridge Studies in Advanced Mathematics, Cambridge University Press, 2011, 2nd Edition.